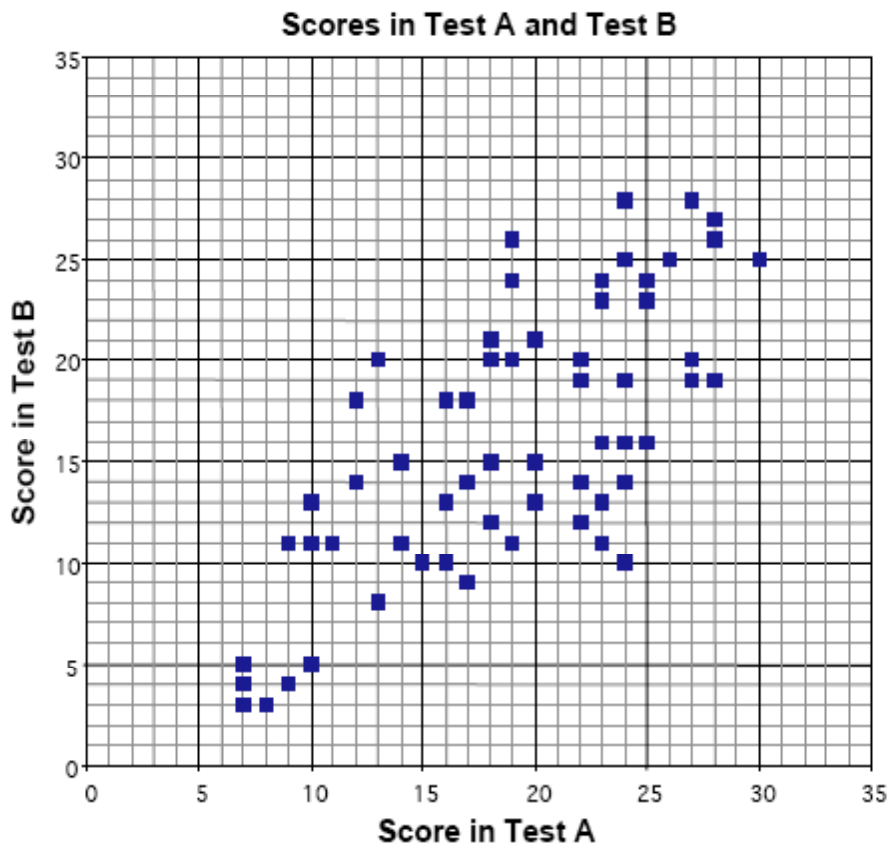

Scatter Diagram

This problem gives you the chance to:

- discuss and understand a scatter plot of real data
-

A group of 66 students took two tests; Test A and Test B.

In the scatter diagram, each square represents one student and shows the scores that student got in the two tests.



1. The mean score for Test A was 19 and the mean score for Test B was 16.

Plot a point to show this on the scatter diagram.

2. Draw a line of best fit on the scatter diagram.

How can a line of best fit be used?

3. Here are five statements about the scores shown on the scatter diagram.

If a statement is true check (✓) it.

If it is not true, write a correct statement.

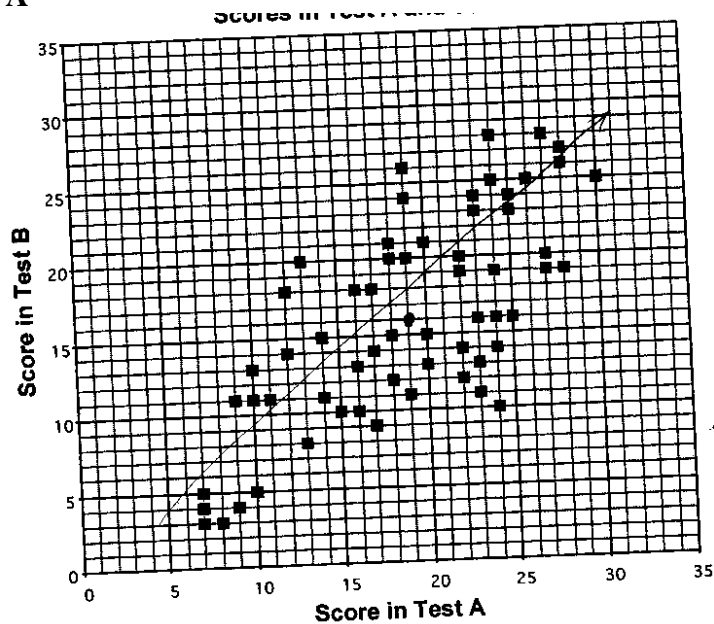
Statement	Check (✓) or write correct statement
The scatter diagram shows positive correlation between the scores on Test A and the scores on Test B.	
The lowest score on Test A is lower than the lowest score for Test B.	
The range of scores on Test B is 25.	
The student with the highest score on Test A also has the highest score on Test B.	
The biggest difference between a student's scores on the two tests is 5.	

Scatter Diagram	Rubric	
<p>The core elements of performance required by this task are:</p> <ul style="list-style-type: none"> • discuss and understand a scatter plot of real data <p>Based on these, credit for specific aspects of performance should be assigned as follows</p>	points	section points
1. Point correctly plotted	1	1
<p>2. Draws a line that best fits the data.</p> <p>Gives a correct statement such as: A line of best fit can be used to estimate a students' score in one test if you know their score in the other.</p>	1 1	2
<p>3. Correctly completes the table such as:</p> <p>√ No. The lowest score on Test A (7) is greater than the lowest score on Test B (3).</p> <p>√ No. The student with the highest score on Test A (30) does not have the highest score on Test B (28).</p> <p>No. The biggest difference is more than 5 (accept 14).</p>	1 1 1 1 1	5
Total Points		8

Looking at Student Work on Scatter Diagrams:

Student A is able to graph the point and draw in a line of best fit. The student understands that best fit shows the relationship between the two variables and the trend of the graph. The student does not mention that the line can be used to make predictions of one score given the other score. The student is able to analyze the statements in the table, determine whether they are accurate, and make appropriate rewrites where needed using information from the scatterplot.

Student A



1. The mean score for Test A was 19 and the mean score for Test B was 16.
Plot a point to show this on the scatter diagram. ✓

2. Draw a line of best fit on the scatter diagram.
How can a line of best fit be used? ✓

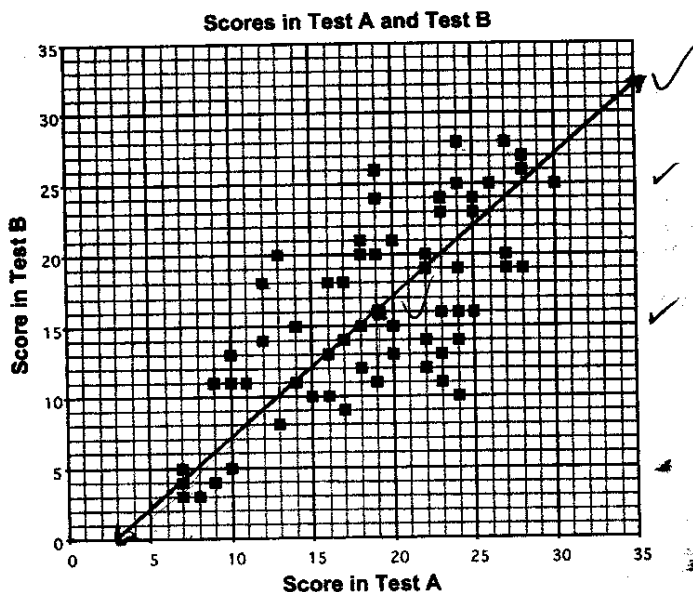
To show the trend and correlation of the scatter diagram. ✓

Student A, continued

Statement	Check (✓) or write correct statement
The scatter diagram shows positive correlation between the scores on Test A and the scores on Test B.	✓
The lowest score on Test A is lower than the lowest score for Test B.	The lowest score on Test B is lower than the lowest score for test A ✓
The range of scores on Test B is 25.	✓
The student with the highest score on Test A also has the highest score on Test B.	The student with the highest score on Test A doesn't have the highest score on Test B ✓
The biggest difference between a student's scores on the two tests is 5.	The biggest difference between a student's scores on the two tests is not 5 ✓

Student B makes the mistake of confusing line of best fit with slope. The student makes the common error of confusing the range of scores with the highest score for test B. The student has trouble with the fourth statement, thinking that we are looking for the highest overall score rather than a comparison of two scores for the same person. Many students realized that the biggest difference in scores was larger than 5, but had difficulty quantifying that value.

Student B



1. The mean score for Test A was 19 and the mean score for Test B was 16
Plot a point to show this on the scatter diagram.

2. Draw a line of best fit on the scatter diagram.

How can a line of best fit be used?

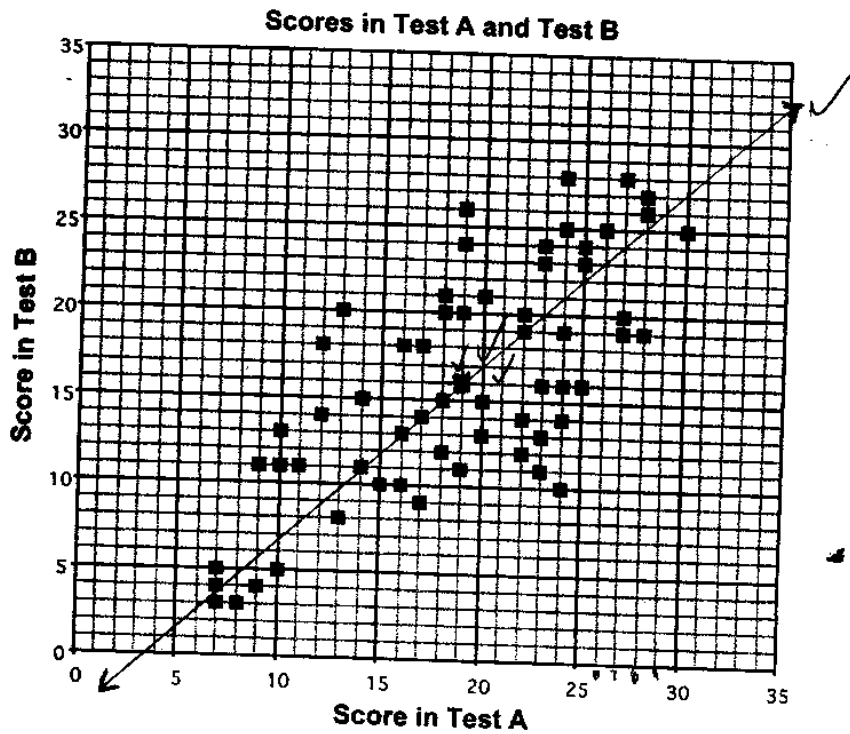
If can help show you the positive slope of the scatter plots.

Student B, continued

Statement	Check (✓) or write correct statement
The scatter diagram shows positive correlation between the scores on Test A and the scores on Test B.	✓ ✓
The lowest score on Test A is lower than the lowest score for Test B.	The lowest score on test B is lower than the lowest score on test A. ✓
The range of scores on Test B is 25.	The range on Test B is 28. X
The student with the highest score on Test A also has the highest score on Test B.	The student scored higher on test B. X
The biggest difference between a student's scores on the two tests is 5.	The biggest difference is 9. X

Student C might be grasping at the idea of trend or correlation, but at the end describes line of best fit as a way of seeing if you did better on one test than the other. Again the student is unable to quantify the range or the exact value of the largest difference between scores on the two tests.

Student C



1. The mean score for Test A was 19 and the mean score for Test B was 16.
Plot a point to show this on the scatter diagram.

2. Draw a line of best fit on the scatter diagram.

How can a line of best fit be used?

To determine if the line is positive or negative
Or to see if you got better test A. or test B + scores

Student C, continued

Statement	Check (✓) or write correct statement
The scatter diagram shows positive correlation between the scores on Test A and the scores on Test B.	✓
The lowest score on Test A is lower than the lowest score for Test B.	The lowest score on Test B is lower than the lowest score for Test A. ✓
The range of scores on Test B is 25.	The range of scores on Test B is 20. ✗
The student with the highest score on Test A also has the highest score on Test B.	The student with the highest score on test A doesn't have the highest score on test B. ✓
The biggest difference between a student's scores on the two tests is 5.	The biggest difference between a student's scores on two tests is 9. ✗

$$\begin{array}{r} 24 \\ -3 \\ \hline 20 \end{array}$$

Student D thinks the line of best fit determines who is above or below average. Many students, especially in middle school confuse line of best fit with average.

Student D

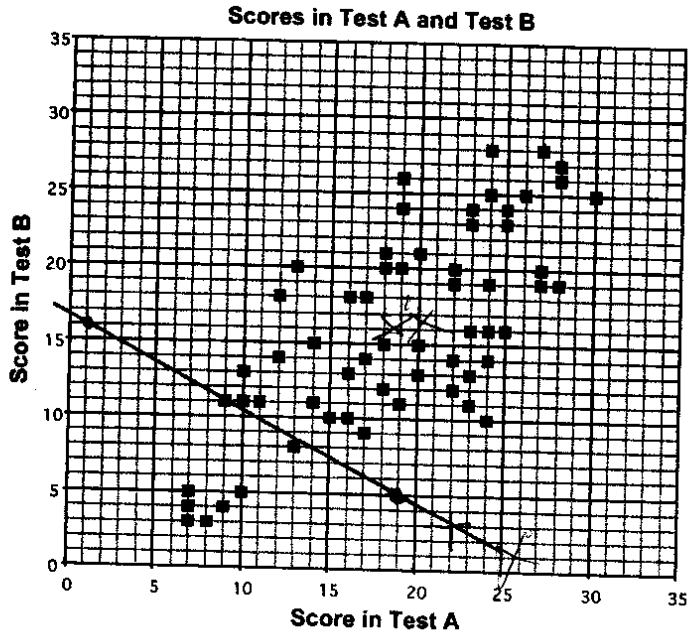
2. Draw a line of best fit on the scatter diagram.

How can a line of best fit be used?

This can be used to determine when the student D gets a score on upper part of the line they are above average, and below is below average. ✗

Student E shows a line of best fit with a negative slope. Notice that the student connects 1,16 with 19,5; drawing two points instead of one to represent 19,16 or the scores for the two tests. The student confuses the range with the highest value on the graph scale rather than a score represented on the graph. The student realizes the fourth statement is incorrect, but cannot rewrite it. It is unclear what the student is thinking about in his final statement. Maybe he notices that some dots are one apart.

Student E



1. The mean score for Test A was 19 and the mean score for Test B was 16.
Plot a point to show this on the scatter diagram.

2. Draw a line of best fit on the scatter diagram.

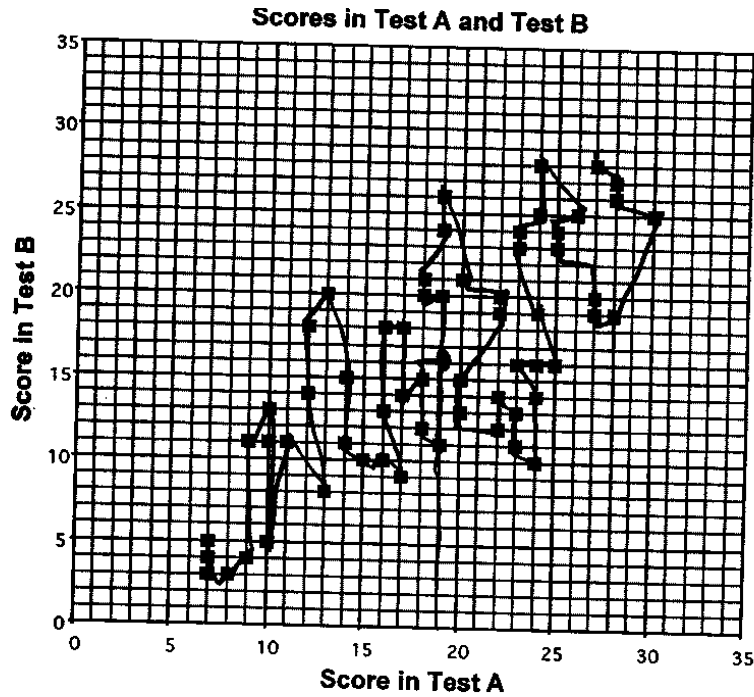
How can a line of best fit be used?

To tell what was the difference

Statement	Check (✓) or write correct statement
The scatter diagram shows positive correlation between the scores on Test A and the scores on Test B.	✓ ✓
The lowest score on Test A is lower than the lowest score for Test B.	✓ ✗
The range of scores on Test B is 25.	no its 35 ✗
The student with the highest score on Test A also has the highest score on Test B.	NO ✗
The biggest difference between a student's scores on the two tests is 5.	No its one ✗

Student F tries to connect all the dots. It is unclear what the student understands because, like a few students, all boxes are checked on the second page of the task.

Student F



1. The mean score for Test A was 19 and the mean score for Test B was 16.
Plot a point to show this on the scatter diagram. *(19, 16)*

2. Draw a line of best fit on the scatter diagram.

How can a line of best fit be used?

To connect all them

Statement	Check (✓) or write correct statement
The scatter diagram shows positive correlation between the scores on Test A and the scores on Test B.	✓
The lowest score on Test A is lower than the lowest score for Test B.	✓
The range of scores on Test B is 25.	✓
The student with the highest score on Test A also has the highest score on Test B.	✓
The biggest difference between a student's scores on the two tests is 5.	✓

Student G recognizes which statements are correct, but cannot figure out how to rewrite the incorrect statements. Like Student G many students did not attempt to rewrite the statements, either because they didn't know how to make the appropriate calculation or were unsure about the meaning of the statement.

Student G

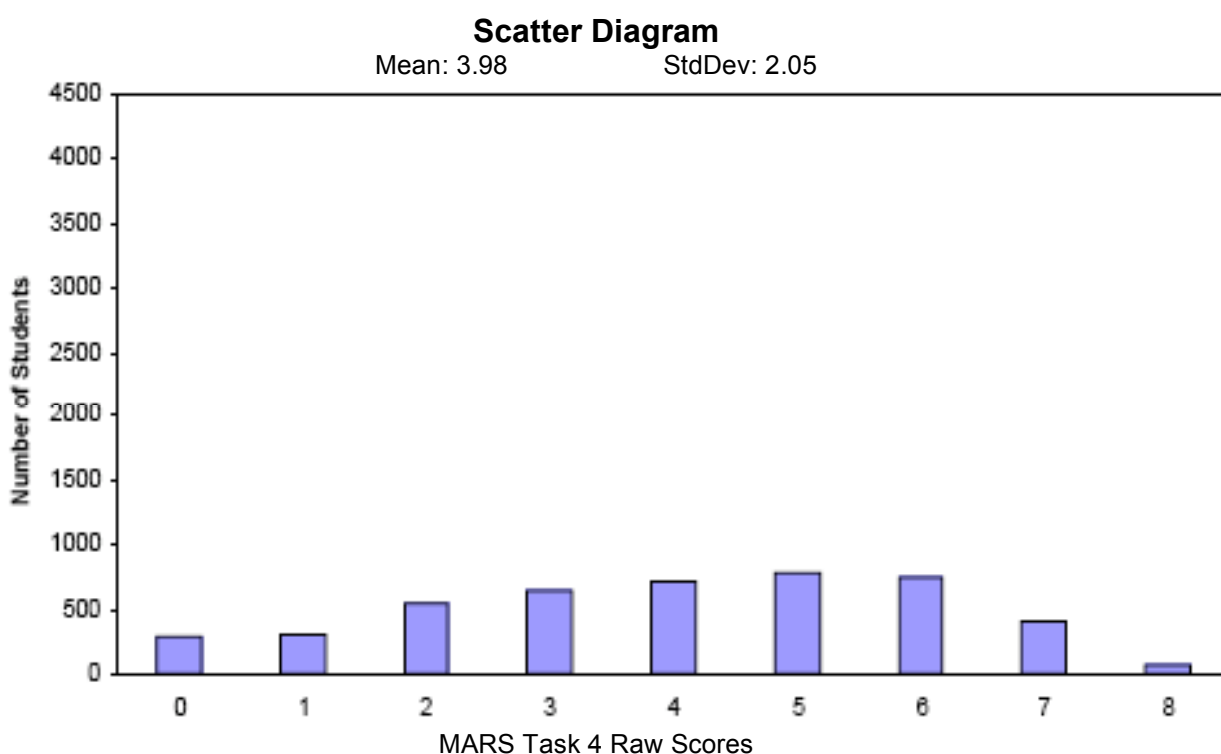
IF IT IS NOT TRUE, WRITE A CORRECT STATEMENT.

Statement	Check (✓) or write correct statement
The scatter diagram shows positive correlation between the scores on Test A and the scores on Test B.	✓
The lowest score on Test A is lower than the lowest score for Test B.	No test b has a lower test score
The range of scores on Test B is 25.	✓
The student with the highest score on Test A also has the highest score on Test B.	
The biggest difference between a student's scores on the two tests is 5.	

✓
✓
✓
X
X

Teacher Notes:

Frequency Distribution for Task 4 – Course 1 - Scatter Diagram



Score:	0	1	2	3	4	5	6	7	8
Student Count	296	307	547	649	712	783	740	408	70
% < =	6.6%	13.4%	25.5%	39.9%	55.7%	73.0%	89.4%	98.4%	100.0%
% > =	100.0%	93.4%	86.6%	74.5%	60.1%	44.3%	27.0%	10.6%	1.6%

The maximum score available for this task is 8 points.

The minimum score needed for a level 3 response, meeting standards, is 4 points.

Most students, about 93%, were able to see that the scatterplot showed a positive correlation between scores on the two tests. Many students, about 74%, were able to plot the point and recognize that statements 1 and 3 in the table were true. More than half the students were able to plot the point, recognize that statements 1 and 3 were correct, and either draw a line of best fit or rewrite statement 2 or 4. About 10% of the students could meet most of the demands of the task including plotting a point, drawing a line of best fit, comparing lowest scores on A and B, finding range for test B, comparing highest scores on A and B, and finding the biggest difference. Only about 2% of the students could explain what the line of best fit represents in terms of the context of the problem. 7% of the students scored no points on this task. Only 13% of those students attempted the task.

Scatter Diagram

Points	Understandings	Misunderstandings
0	<u>20% attempted the task.</u> <i>14% attempted the task.</i>	Most students were unwilling to attempt the task. Perhaps they are unfamiliar with this type of graph.
1	Students recognized that the graph showed a positive correlation.	Many students thought that the lowest score on A is lower than the lowest score on B (<u>22%</u> <i>38%</i>).
2	Students could recognize the positive correlation and knew the range was 25 for Test B.	About 12% thought the range was 28. Other common values for range were 20 and 30.
4	Students could identify positive correlation, range, plot a point on a scatter diagram, and draw a line of best fit.	<u>13.5%/30%</u> did not attempt to plot the point. Often the numbers were plotted backwards (16,19) instead of (19,16). <u>19%/27%</u> did not attempt a line of best fit. About 6% of the students drew lines of best fit that were too high. About 4% drew lines with a negative slope.
6	Students can plot a point, draw a line of best fit, recognize correlation and range, correct a statement about lowest scores and highest scores on both tests.	Students could not identify the biggest difference between a student's score on both tests. <u>5%/ 22%</u> did not attempt this part of the task. <u>38%/45%</u> thought that 5 was the biggest difference. The most common error choices were 7 and 9. Students could see that the difference was larger than 5, but could not find the exact value.
7		Students could not explain what the line of best fit would be used for. <u>18.5%/27%</u> did not attempt this part of the task. <u>40%/ 12%</u> thought it represented the average score. About 5% thought it was to show the slope. Many answers related to real life: who did the best or worst, show a trend, where students are in the class.
8	Students can plot a point, draw a line of best fit, recognize correlation and range, correct a statement about lowest scores and highest scores on both tests. Students could also identify the purpose of the line of best fit.	

Underlined statements indicate an error patterns found more exclusively in middle school. Italic statements indicate an error pattern found more exclusively in high school papers.

Based on teacher observation, this is what algebra students knew and were able to do:

- Plot a point
- Draw a line of best fit
- Identify statements about the graph which were correct
- Identify statements about the graph which were incorrect

Areas of difficulty for algebra students:

- Rewriting incorrect statements to make true statements about the graph, particularly the two lowest scores, the highest scores, and the largest difference in scores
- Understanding the purpose of the line of best fit

Questions for Reflection on Scatter Diagram

- What types of experiences do your students have with analyzing data? Are they familiar with scatter plots?
- Were your students able to graph the point and approximate the line of best fit?
- Could your students identify the two correct statements in the table?
- Were students able to see what was true about the lowest scores? The highest scores?
- What did you students seem to think the line of best fit represented? Why do you think this was so difficult?

Look at student work on purpose of line of best fit. How many of your students:

Predict values	No response	Average	Slope	Details about test scores/ e.g who is the highest	Other

- How can you design a lesson to help confront these issues? What does it show that students don't understand about average?
- Do you think your students understand positive and negative correlation? What further questions might you want to ask students to probe their understanding on a deeper level?

Implications for Instruction

Students at this grade level should be familiar with a variety of ways to display and interpret data. When given a set of data, they should be comfortable picking the display that best represents that data or that suits the purpose for answering the questions for which the data was collected. Giving students opportunities to pose questions and collect their own data helps make the choice of data display selection seem more relevant and clarifies the purpose of each choice.

Students should see that scatter plots help to answer questions about the relationship between two items, is there a cause and effect? Does one seem to help predict the other? Is there a relationship between shoe size and weight? Shoe size and speed running a 50 yd. dash? The amount of homework completed and test scores?

Knowing students ideas about average as the most common misinterpretation of tests scores, the teacher might plan a lesson on returning the papers to help students confront this misconception. The teacher might ask them to talk about what the average is for each test, what the average might mean for one particular student on the graph, say point (24,10) or (19,26). Where would you plot a point to represent each average? If you started to graph all the points, what would you notice about the plots of the averages? Now have students find the number of points and try to design a line dividing the points into equal groups above and below the line? How does this line look different? Which line is the best predictor of someone's score? Test out several cases. Then have the students examine and think about these same questions using some other scatter plots. Students should be exposed to several that have different correlations, positive, negative, no correlation, tight correlation, and loose correlation. Then students should be able to answer why this is a useful tool for researchers. Students should also be given situations that appear to have a correlation, but don't have a cause and effect relationship, where the results are just coincidental. It is in unpacking these types of discrepancies that the mathematics becomes engaging for students and the ideas start to take on meaning.

Teacher Notes:

Performance Assessment Task				
Scatter Diagram				
Grade 9 task aligns in part to CCSSM grade 8				
This task challenges a student to select and use appropriate statistical methods to analyze data. A student must be able to understand the relationships between two sets of data to describe trends and shape of the plot including correlations and lines of best fit. A student must make and critique inferences based on the data and evaluate the validity of drawn conclusions.				
Common Core State Standards Math - Content Standards				
Statistics and Probability				
Investigate patterns of association in bivariate data.				
8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and non-linear association.				
8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.				
Common Core State Standards Math – Standards of Mathematical Practice				
MP.3 Construct viable arguments and critique the reasoning of others.				
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.				
MP.4 Model with mathematics.				
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.				
Assessment Results				
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.				
Grade Level	Year	Total Points	Core Points	% At Standard
9	2005	8	4	60%